Central European Journal of Energetic Materials, **2012**, *9*(4), 353-364
ISSN 1733-7178

Mathematical Formulation and Validation of Muraour's Linear Burning Rate Law for Solid Rocket Propellants

Himanshu SHEKHAR

High Energy Materials Research Laboratory (DRDO), Sutarwadi, Pune-411 021, India
E-mail: himanshudrdo@rediffmail.com

Abstract: Linear variation of burning rate with pressure (burning rate, r = H + Sp), referred in the literature as Muraour's law, is adopted as the burning rate law for solid rocket propellants. The two constants 'H' and 'S' are the vacuum burning rate and the slope of burning rate variation, respectively. The conventional power law of the burning rate, $r = ap^n$, is also analyzed and its practical, anomalous behaviour such as zero burning rate at zero pressure, the reduction in pressure sensitivity of the burning rate at higher pressures, the lower burning rate for the high pressure index in typical situations etc, are explained with illustrations. Like the conventional power law of burning rate, the linear burning rate law considered here is also empirical but mathematically simpler than the power law. Using burning rate and pressure data from various literature sources similar regression coefficients are observed for the conventional power law as well as for the alternative linear burning rate law. The mathematical concept for the evolution of the pressure time profile with the considered linear burning rate law is developed and validated practically with the actual firing of rocket propellants as uninhibited, tubular configurations in a ballistic evaluation motor (BEM). Close matching of the firing curve, predicted by the conventional power law and by the proposed linear burning rate law validates the mathematical formulation. The considered linear burning rate law is simple, easy to apply and gives a better representation of the burning rate behaviour of solid rocket propellants.

Keyword: rocket propellants, burning rate, pressure index, ballistics prediction, Muraour's law

Introduction

Solid rocket propellants are used in missiles, rockets and launch vehicles for their propulsion, by rapid expansion and discharge of the gaseous products of combustion. The burning rate equation for conventional solid rocket propellants is Vieille's law, where the burning rate (r) changes with pressure (p) raised to the pressure index (n). For stable and steady combustion, the value of the pressure index (n) for solid rocket propellants is between 0 and 1. If the value of the pressure index (n) is very close to 0, the burning rate becomes independent of pressure. On the other hand, if the value of the pressure index (n) reaches or exceeds 1, the rocket motor operation becomes unstable. In Brassey's Series Vol 2 [1], the value of the pressure index is stated to be around 0.8 to 0.9 for gun propellants and around 0.3 for rocket propellants. This law is empirical in nature and was adopted due to its simplicity as compared to the phenomenological burning rate law proposed earlier by Summerfield et al. [3]. However, in the operating pressure range, this conventional burning rate law suffers from anomalous behaviour. Consequently, an attempt is made in this paper to explore the applicability of Muraour's linear burning rate law [2]. Although this law is reported to be useful above 200 atmospheres pressure, in the present article this law is used as an alternative to the prevailing Vieille's burning rate law in the normal operating pressure range of rockets (3 to 10 MPa). The conceptual evolution of the linear burning rate law, the mathematical formulation for the prediction of the pressure time profile, the analysis of the burning rate law and its practical validation from the actual firing of a rocket motor is described in this article.

Concept Evolution

The burning rate law for the combustion of a solid rocket propellant is stated to be Vieille's law (Equation 1), where the burning rate (r) is empirically expressed as being proportional to the pressure (p) raised to the pressure index (n). The pressure index (n) of the burning rate is stated to be independent of pressure (p) and temperature (T). The constant of proportionality (a) for this law is said to be dependent on the temperature (T) of the propellant during burning. The conventional power law for the variation of burning rate with pressure for solid rocket propellants is stated below.

Burning rate, $r = a\ p^n$ (1)

This is a universally accepted expression as the burning rate law for solid rocket propellants. The universal acceptability of this law is clear from the fact that the pressure index is indicated in many research articles for rocket propellant formulations. Previously Summerfield's equation (Equation 2), derived from the granular diffusion flame (GDF) model, was proposed and used as the burning rate equation for composite solid propellants [3].

$$\mathbf{1/r = \alpha/p + \beta/p^{1/3}} \tag{2}$$

where α is a constant called the gas phase reaction time parameter and β is another constant called the diffusion time parameter. This equation was evaluated and several unexplained phenomena and deviations were enumerated in later research [4]. Due to the complexity and difficulty in applying Summerfield's equation for predicting the performance of rocket motors, the power law has been accepted as the burning rate behaviour of solid rocket propellants. Research on solid rocket propellants invariably mentions the pressure index as one of the properties of the propellant [5-8]. The values of the coefficients in the burning rate power law are determined by firing propellants in ballistic evaluation motors (BEM) [9, 10]. In one of the considered papers, it is stated that a low pressure index at low pressures exhibits a transition to a high pressure index in the pressure range 20 to 40 MPa [11]. Raman et al. [12] presented the variation of the burning rate with pressure for many propellant formulations in the pressure range 3.5 to 10.5 MPa. These data are presented in Figure 1 for some of the compositions. The curve is not on a logarithmic scale and the variation of the burning rate with pressure seems to be linear.

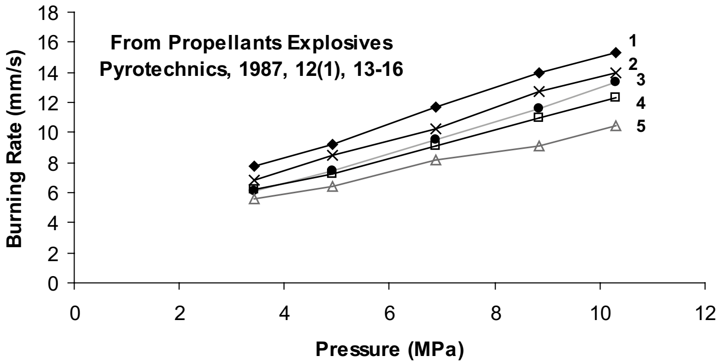

Figure 1. Burning rate variation from a paper published in 1987 [12].

Xu Li-hua et al. [13] have presented burning rate data for 5 batches of propellant at 5 different pressures and have calculated the pressure index of the burning rate for each solid rocket propellant formulation. The curves are shown in Figure 2 and have a linear variation of burning rate with pressure.

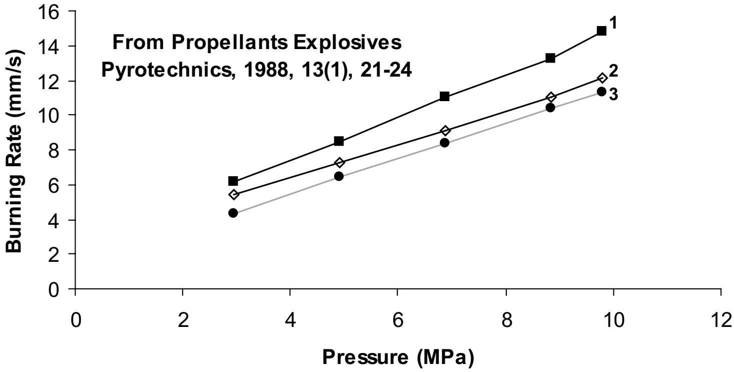

Figure 2. Variation of burning rate with pressure from a paper in 1988 [13].

Similar burning rate versus pressure data for different types of propellant are given in the literature [14-16] and close scrutiny of these gives a definite linear variation of burning rate with pressure.

In another paper [17], burning rate versus pressure data are not provided, but the burning rate law for different propellant formulations is described. It is obvious that, the pressure index is not independent of temperature and of different formulations; the variation of the pressure index 'n' with increase in temperature is inconsistent. An increase in temperature reduces the pressure index for some formulations, but for some other formulations the reverse trend is also observed. The burning rate coefficient 'a', which is supposed to increase with an increase in temperature, is found to decrease for some formulations, indicating a propellant with a negative temperature sensitivity coefficient. For one of the compositions in the paper, separate burning rate laws are provided for different pressure ranges. At pressures higher than 3.5 MPa, a burning rate law with a higher pressure index is specified. So the burning rate law, which is fitted to the experimental data, does not represent the burning rate variation of solid rocket propellants with pressure. In other reports [18, 19], a power law of burning rate is used.

The power law of burning rate variation with pressure does not represent propellant behaviour correctly, due to following observations derived from various considered papers:

1. The burning rate law is a mathematical convenience for ease of internal

ballistic calculations and the depicted constants in any burning rate law are not a measurable parameter for solid rocket propellants. The burning rate and pressure are the measured parameters.

2. In the operating pressure ranges, when provided, a linear variation of burning rate with pressure is observed.
3. Two different burning rate laws for the same propellant in different pressure regimes is not a valid and convenient method for performance prediction.
4. Although the power law is well developed for performance prediction of solid rocket propellants, a similar approach can be developed for the linear burning rate law.

Hence it is proposed to represent the variation of the burning rate of solid rocket propellants with pressure as a linear law.

Calculation Strategy

The slope of the burning rate with pressure reduces as the pressure rises. The curve for the burning rate variation with pressure has a high variation of slope for lower pressure, and at higher pressures it becomes almost a straight line. The equation of the slope is given below (Equation 3).

Slope of burning rate variation curve, dr/dp = nr/p (3)

For most of the applications, the burning rate of a solid rocket propellant lies between 8 and 35 mm/s at a pressure of 7 MPa. This is reflected in the literature cited in this article. It is also observed that the value of the pressure index 'n' varies from 0.1 to 0.7. Obviously, for low pressure values, the slope of the burning rate variation with pressure may be large, but at moderate pressures (say above 2 MPa), the numerical value of the slope reduces drastically and the curves resemble a straight line. In addition to this, if the propellant has a low pressure index, linearity is observed at much lower values of the pressure. In view of the practical data available in various literature sources and the mathematical nature of the power law, we propose that the variation of the burning rate of solid rocket propellants with pressure may be represented by a linear law (Equation 4).

Burning rate, r = H + Sp (4)

where H is the vacuum burning rate and S is the slope of the burning rate with pressure (dr/dp).

The law was applied to the data of burning rate variation with pressures from all the cited literature and was found to give a better (or comparable) regression coefficient than the power law. A summary of the burning rate laws and the regression coefficients for the two figures are tabulated in Table 1. The laws are valid for the pressure range 4 to 10 MPa.

Table 1. Comparison of the power law and the linear burning rate law

Fig. No.	Curve No.	Power burning rate law			Linear urning rate law		
		a mm/s/MPa^n	n	R^2	H (mm/s)	S (mm/s/MPa)	R^2
1	1	3.4916	0.6315	0.9933	3.8785	1.1244	0.9971
1	2	2.9845	0.6579	0.9942	3.2031	1.0539	0.9967
1	3	2.483	0.709	0.9943	2.4062	1.0476	0.999
1	4	2.7283	0.6367	0.9877	2.9246	0.908	0.9975
1	5	2.6914	0.5715	0.9838	3.1031	0.7073	0.9895
2	1	2.8036	0.7157	0.9951	2.5012	1.2351	0.9983
2	2	2.5878	0.6646	0.9956	2.5366	0.9659	0.9994
2	3	1.7936	0.8047	0.9997	1.3476	1.0212	0.9996

Even for the worst case, data from reference 16 , where a low burning rate (r) propellant is studied and limited data are made available, the regression coefficients match. The linear burning rate law has the following advantages over the conventional power law of burning rate:

1. The power law indicates that the propellant burning rate is zero at zero pressure, which is not true. Propellant can burn in open conditions and also in a vacuum. The linear burning rate law takes care of this through its first term 'H', which indicates the vacuum burning rate of the propellant.
2. At low pressure, the power law indicates a rapid variation in burning rate with pressure and at high pressures, a slow variation of burning rate with pressure is depicted. However, the burning rate variation of solid rocket propellants at high pressures is higher. Contrary to this, the linear law does not differentiate between high and low pressures and gives the same slope for all pressure conditions.
3. Non-linearity in burning rate variation with pressure exists at low pressures (< 2 MPa), which is not the operating pressure domain of rocket motors. This is below the low pressure combustion limits of double base as well as composite propellants. Since non-linear variation of the burning rate of propellants with operating pressure is excluded from the practical operational domain of rockets, the importance of the power law in the practical domain is only superficial.

4. Estimation of the burning rate by the power law at pressures higher than the actual measured domain, generally gives a lower value than that observed. The power law gives a slope reduction with increase in pressure, but the burning rate of rocket propellants actually exhibits the reverse trend.
5. The dimensions of 'a' is very complex in the power law. The other parameter, the pressure index 'n', is dimensionless. In the linear burning rate law, the vacuum burning rate 'H' has the same dimensions as the burning rate and the slope 'S' has the dimensions of burning rate per unit pressure.
6. The linear burning rate law of Muraour considered here can be used for the prediction of the pressure-time profile of rocket motors with the same ease as Vieille's power law.

For the calculation of pressure using the power law, the mass balance (mass generated in the rocket motor chamber by combustion of propellant = mass discharge through the nozzle) equation is used and the chamber pressure is given by the following relation for the steady state condition (Equation 5).

$$\textbf{Pressure, } \mathbf{p = [(a\rho C^*A_b)/A_t]^{1/(1-n)}} \qquad (5)$$

where ρ = density of the propellant, C* = characteristic velocity of the propellant combustion, A_b = burning surface area of the propellant, A_t = throat area of the nozzle.

A similar mass balance equation can be used with the linear burning rate law and the pressure-time profile can be predicted for the steady state condition. The equation for the pressure from the mass balance of combustion gases with the linear burning rate law is given below (Equation 6).

$$\textbf{Pressure, } \mathbf{p = H / [\{A_t/(\rho C^*A_b)\}-S]} \qquad (6)$$

Analysis and Experimental Validation

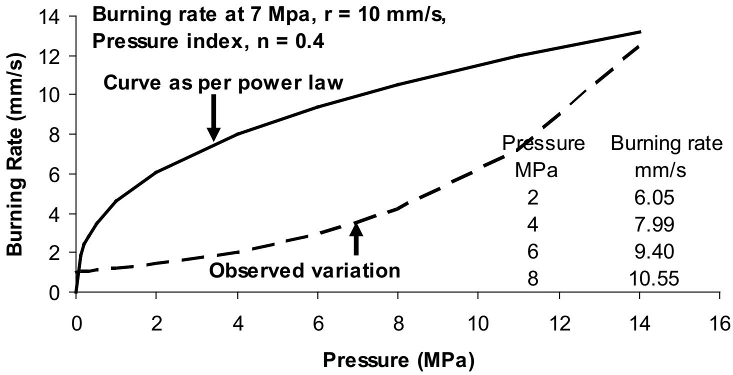

Figure 3. Nature of the conventional power law of burning rate.

When the power law is plotted (Figure 3) on the pressure-burning rate plane, it is observed that at high pressures the curve becomes flat (lower slope or close to horizontal). This indicates a relatively small change in burning rate of propellant at higher pressures for a given change in pressure than that at lower pressures. The increase in the burning rate between 2 and 4 MPa is larger than between 6 and 8 MPa (burning rate change 2 vs ~1.1 mm/s). This variation is also depicted in the Figure, which resembles Figure 7 of an article published in 2007 [20]. However, the actual behaviour of the propellant is the reverse of this. To tackle this anomaly, several papers indicated higher pressure index values at higher pressures [11, 18]. The actual nature of the burning rate variation with pressure should have a rising slope, while the situation depicted by the power law is the reverse. So, the conventional power law is mathematically not competent to represent the variation of burning rate with pressure. As an alternative, a linear burning rate law is considered and its mathematical formulation for the calculation of the operating pressure in a rocket motor under similar conditions has been developed in previous section.

Another peculiar situation is observed when the pressure index ‘n’ is changed for the same burning rate at a reference pressure. A higher pressure index should represent higher pressures, but Figure 4 depicts an anomalous situation. For reference, the burning rate at 7 MPa is taken as 10 mm/s and the pressure index is changed from 0.2 to 0.6 in intervals of 0.1. It is clear that at pressures less than the reference pressure (7 MPa), a higher pressure index ‘n’, results in a lower burning rate. In addition to this, a lower pressure index ‘n’, at low pressures, gives a faster change in burning rate with pressure, but the burning rate variation with pressure becomes almost linear for pressures higher than 2 MPa. A lower pressure index 'n' indicates a lower slope (S) in the linear burning rate law and a large vacuum burning rate (H).

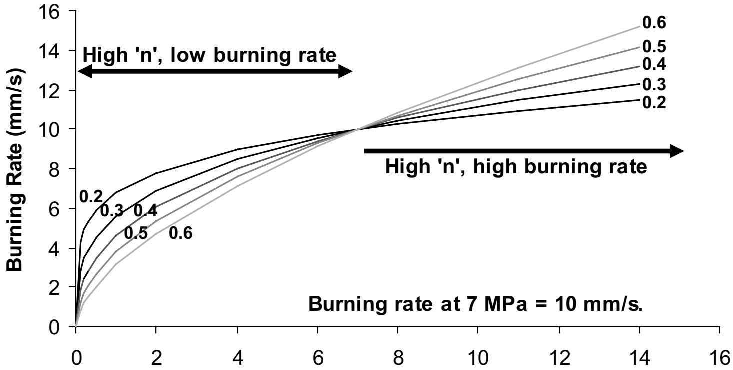

Figure 4. Variation of burning rate for different pressure indices.

The developed mathematical formulation (equations 5 and 6) is validated using firing data of uninhibited, tubular propellant grains fired in ballistic evaluation motors (BEM) and generating a regressive firing curve. One such firing curve (Figure 5) is appended as an example. The tubular, solid propellant grain, with an outer diameter (D) of 115.3 mm, an inner diameter (d) of 59.46 mm and length (L) of 199.5 mm, was evaluated in a ballistic evaluation motor. The pressure (in kg/cm^2) -time (in seconds) profile, as received from the data acquisition system of static firing, is given. The non-aluminized, composite propellant used had a density of 1679 kg/m^3 and a characteristic velocity of 1501 m/s. The throat diameter in the firing was 20.1 mm. In the static evaluation, propellant was consumed and the consumption was expressed in terms of web consumed. Web burnt (w) alters the burning surface area of the propellant. During firing, the outer diameter and the length of the propellant grain reduces, whilst the inner diameter increases. These parameters are expressed in terms of web burnt (w). For the calculation of the pressure-time profile, the burning surface area of the tubular propellant with web burnt (w) is calculated by the equation given below (Equation 7).

$$\textbf{Burning area, } A_b = (\pi/2) \times (D+d) \times (2L + D - d - 8w) \qquad (7)$$

At the beginning the web burnt is zero and is increased in small steps until the complete web of the propellant is consumed. For the calculated burning surface area, the pressure was calculated by both the power law and the linear burning rate law, using the formulae given in the previous section. For the pressure calculated, the instantaneous burning rate is calculated from the respective burning rate laws. For each part of extra web burnt, the time elapsed was calculated. This calculation is repeated until the complete web of the propellant is consumed. The maximum web of the propellant in the tubular configuration is given by the following relation (Equation 8).

$$\textbf{Maximum web, } w_{max} = (D-d)/4 \tag{8}$$

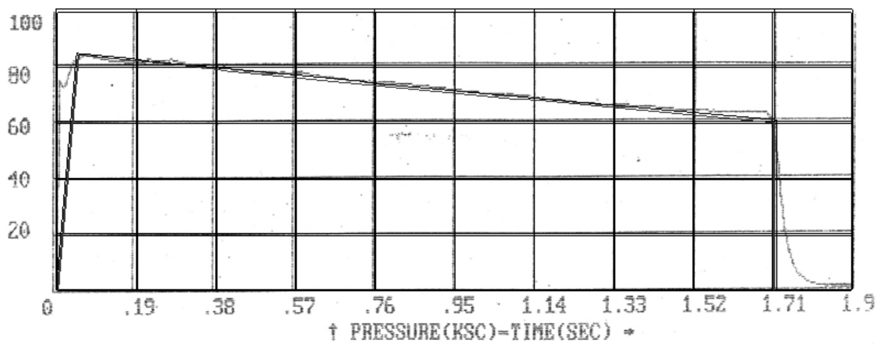

Figure 5. Pressure-time profiles superimposed on the actual BEM firing curve.

The pressure-time profiles, as calculated by the power law and the linear burning rate law, are superimposed on the actual firing curve. The ignition transient was not analyzed and an ignition delay of 50 milliseconds was applied in the calculations for both burning rate laws. Furthermore a steady state condition was presumed to exist. The pressure-time profiles as calculated from the power law and the linear burning rate law match each other and also the actual firing curve. This validates the linear burning rate law for solid rocket propellants and also ensures there is no degradation in calculated performance when the existing power law is replaced with the linear burning rate law.

Conclusion

The burning rate of a solid propellant is generally called the linear rate of burning, but it is expressed in conventional literature as a power law. With Muraour's linear burning rate law, the burning rate law of a propellant becomes a linear burning rate law in the true sense. Apart from imparting simplicity, the parameters of the linear burning rate law can be directly assigned physical significance. The burning rate, r, is represented by H + Sp, where 'H' is the vacuum burning rate and 'S' is the slope of the pressure-burning rate curve. The developed correlation for the prediction of the pressure-time profile using the linear burning rate law is validated by static evaluation of an uninhibited, tubular propellant grain in a ballistic evaluation motor (BEM). The exact matching validates the linear burning rate law formulation and the developed ballistic calculation strategies.

Symbols used:

β – Constant in Summerfield's burning rate law (diffusion time parameter)
α – Constant in Summerfield's burning rate law (gas phase reaction time parameter)
ρ – Density of the propellant
a – Burning rate coefficient in Vieille's law
A_b – Burning surface area of the propellant
A_t – Throat diameter of the nozzle of the rocket motor
C* – Characteristic velocity of the propellant
g – Conversion factor of mass into weight
H – Vacuum burning rate of solid rocket propellant
n – Pressure index in Vieille's law
p – Pressure
r – Burning rate of the solid rocket propellant
S – Rate of change of burning rate with pressure for the solid rocket propellants (dr/dp)

References

[1] Bailey A., Murray S.G., *Explosives Propellants and Pyrotechnics*, Vol. 2, Brassey's, UK, **1989**.
[2] Williams F.A., Barrere M., Huang N.C., *Fundamental Aspects of Solid Rockets*, Technivision Services Slough, England, October **1969**, pp. 192-193.
[3] Summerfield M., Sutherland G.S., Webb M.J., Taback H.J., Hell K.P., *Burning Mechanism of Ammonium Perchlorate Propellants*, ARS Preprint, **1958**, pp. 737-58.
[4] Blair D.W., Bastress E.K., Hermance C.E., Hall K.P., Summerfield M., Some Research Problems in the Steady-State Burning of Composite Solid Propellants, *Solid Propellant Rocket Research Conference*, Princeton University, New Jersey, 28-29 January, **1960**.

[5] Kubota N., Aoki I., Burning Rate Characterization of GAP/HMX Energetic Composite Materials, *Propellants Explos. Pyrotech.*, **2000**, *25*(4), 168-171.
[6] Babuk V.A., Dolotkazin I.N., Glebov A.A., Burning Mechanisms of Aluminized Solid Rocket Propellants Based on Energetic Binders, *Propellants Explos. Pyrotech.*, **2005**, *30*(4), 281-290.
[7] Guo X., Li F., Song H., Liu G., Kong L., Li M., Combustion Characteristics of a Novel Grain Binding High Burning Rate Propellant, *Propellants Explos. Pyrotech.*, **2008**, *33*(4), 255-260.
[8] Pande S.M., Sadavarte V.S., Bhowmik D., Gaikwad D.D., Singh R.V., Singh H., Burning Rate – Pressure Relationship of NG-PE-PCP-based High Energy Propellants, *Propellants Explos. Pyrotech.*, **2012**, *37*(2), 241-245.
[9] Anon, *Solid Rocket Motor Performance Analysis and Prediction*, NASA Report SP-8039, May **1971**, N72-18785.
[10] Fry R.S., *Solid Propellant Subscale Burning Rate Analysis Methods for U.S. and Selected NATO Facilities*, The John Hopkins University, Chemical Propulsion Information Agency, Columbia, Maryland, Jan **2002**.
[11] Cole R.B., *Burning Rates of Solid Composite Propellants at Pressures up to 20000 psig (U)*, AD375686, Report No S-80, 23 Sept **1966**.
[12] Raman K.V., Singh H., Rao K.R.K., Ballistic Modification of CMDB Propellants Containing Ammonium Perchlorate, *Propellants Explos. Pyrotech.*, **1987**, *12*(1), 13-16.
[13] Xu L.-H., Wang M.-X., Wang E.-P., Tan H.-M., Chen B.-R, The Properties of 1,3,3,5,7,7-hexanitro-1,5-diazacyclooctane (HCO) and Its Application in Propellants, *Propellants Explos. Pyrotech.*, **1988**, *13*(1), 21-24.
[14] Leu A.-L., Yeh T.-F., Chang F.-M., Liu C.-S., Huang C.-C., Liu F., Shih Y.-S., Burning Behaviour of Composite Solid Propellant Containing Porous Ammonium Perchlorate, *Propellants Explos. Pyrotech.*, **1989**, *14*(3), 108-112.
[15] Kentgens H., Mackowiak H.P., Schoöffl R., Short Action Solid Rocket Motors with Double-Base Propellants, *Propellants Explos. Pyrotech.*, **1996**, *21*(3), 118-126.
[16] Ghorpade V.G., Dey A., Jawale L.S., Kotbagi A.M., Kumar A., Gupta M., Study of Burn Rate Suppressants in Ammonium Perchlorate Based Composite Propellants, *Propellants Explos. Pyrotech.*, **2010**, *35*(1), 53-56.
[17] Oyumi Y., Inokami K., Yamazaki K., Matsumoto K., Burning Rate Augmentation of BAMO based Propellants, *Propellants Explos. Pyrotech.*, **1994**, *19*(4), 180-186.
[18] Miller M. S., *Burning-Rate Models and Their Successors: A Personal Perspective,* US Army Research Laboratory, ARL-TR-2996, June **2003**.
[19] Arvanetes J.C., *Design and Implementation of an Emission Spectroscopy Diagnostic in a High-Pressure Strand Burner for the Study of Solid Propellant Combustion*, M.Sc. Thesis, Department of Mechanical, Materials, and Aerospace Engineering, University of Central Florida, Orlando, Florida, **2006**.
[20] McBurnett M., The Use of Flight Motors in Solid Rocket Propellant Characterization Experiments, *Rocket Magazine*, **2007**, *2*(5), 8-12.